Text

Description automatically generated with medium confidence

1. ( ).
2. , ( )
3. . (.

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Original: , . . ( = = ) ( = )

Converse: : , . . ( = ) ( = = )

Proof original :

Let x and y be arbitrary and fixed elements of the set of integers. Suppose that x and y are odd.

If x and y are odd, by definition of an odd integer, we can write them as and where and are some integer

Then we can write the product of = ).

Using distributive laws, we can write the product of =

Factoring and re-arranging the equation gives =

Therefore, the product of is in the form where is an integer such that .

Thus, if x and y are odd integers then their product is an odd integer.

Proof Converse:

To prove the converse, we will use the contrapositive of the converse,

Let x and y be arbitrary and fixed elements of the set of integers.

Suppose that x is even, or y is even.

Case 1: is even and is odd.

If is even it can take the form 2 where is some integer. If is odd it can the form where is some integer.

We can factor such that which by definition of even integers is even where .

Case 2: is odd and is even

If is odd, it can take the form where is some integer. If y is even, it can take the form where is some integer.

We can factor such that which by definition of even integers is even where .

Therefore, if x is an even integer or y is an even integer then their product must be even.

Thus, it is only the case that for the product of two integers to be odd both integers must be odd.

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Negation: . . (

Proof: Original statement

Let n be an arbitrary and fixed value of the set of positive integers.

Suppose since then

Since then

Furthermore since then

Take x = , then . Therefore, for every that is a positive integer we can find an that is a real positive number such that .

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Proof: (First)

Let be an arbitrary and fixed element of the set of natural numbers.

Suppose that is odd.

Since is odd it can be represented as or for some integer .

Case 1:

Then can be represented as .

Distributing the square gives .

Combining like terms gives .

Factoring gives

Which is divisible by 8.

Case 2:

Then can be represented as .

Distributing the square gives .

Combining like terms gives .

Factoring gives

Which is divisible by 8.

Thus .

(Secondly )

To prove the converse, we will use the contrapositive of the converse,

Let be an arbitrary and fixed element of the set of natural numbers.

Assume is even.

If is even, then for some integer .

Then, can be represented as .

Distributing the square gives

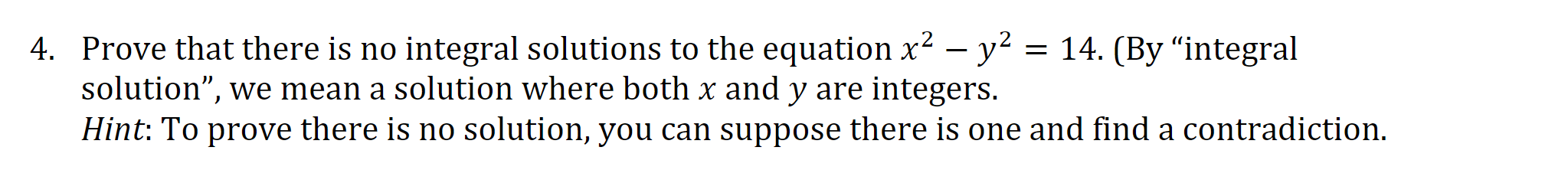
Factoring gives

Let for some integer .

Thus is odd by definition of odd integers. Which is not divisible by 8.

Therefore if is even, then is not divisible by 8.

Thus .



Let and be fixed elements of the set of integers. Assume for the sake of contradiction that - = , and that and are integers.

Solving for y yields:

=

For y to be a integral solution Therefore is bounded by the inequality .

Therefore, the only values of that can satisfy the inequality are and 0.

Case 1:

The is not an integer thus it cannot be an integral solution.

Case 2:

The is not an integer thus it cannot be an integral solution.

Case 3:

The is not an integer thus it cannot be an integral solution.

Case 4:

The is not an integer thus it cannot be an integral solution.

For every integral solution there is no integral solution for which contradicts the assumption that both and can be represented as an integral solution.

Therefore, there are no integral solutions for - = .

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Let x be an arbitrary and fixed element of .

def of intersection.

def of union.

def of compliment.

def of distributive laws.

def of difference.

def of .